EPJ manuscript No.
(will be inserted by the editor)

Comment on "Experimental determination of KPZ height-fluctuation distributions" by L. Miettinen et al.

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Received: March 13, 2012

Abstract. Miettinen *et al.* [Eur. Phys. J. B **46**, 55 (2005)] analyzed height fluctuations of growing interfaces accompanying slow combustion of paper, which had been previously shown to exhibit scaling exponents of the Kardar-Parisi-Zhang (KPZ) class, and claimed that the measured distribution functions agree well with theoretical predictions made for solvable models in the KPZ class. Although their results are encouraging, here I argue that their analysis is not sufficient to make a significant distinction among different distributions discussed in this context. An alternative method is proposed that is capable of testing whether the universal distribution functions of the KPZ class indeed emerge in this slow combustion experiment.

As a series of experimental studies on growing interfaces in slow paper combustion [1,2], which have clearly shown scaling exponents of the Kardar-Parisi-Zhang (KPZ) universality class [3], Miettinen *et al.* measured distributions of interface height fluctuations for globally flat interfaces [4]. Fitting experimentally measured probability density functions (pdfs) by theoretically expected curves, shifted and scaled arbitrarily, they claimed that the interface fluctuations in the transient regime obey the largest-eigenvalue distribution of random matrices in Gaussian orthogonal ensemble (GOE), or the GOE Tracy-Widom distribution [5], and those in the stationary (or saturated) regime obey the distribution called F_0 [6], as predicted theoretically in Ref. [7].

This is a very important investigation, especially in view of remarkable ongoing progress in theory since the last decade on the universal distributions of the growing interfaces in the KPZ class (for recent reviews, see [8–10]). In addition to the above-mentioned distributions for (on average) flat interfaces, theoretical studies on solvable models [7–10] showed in the case of curved interfaces the largest-eigenvalue distribution for Gaussian unitary ensemble (GUE), or the GUE Tracy-Widom distribution, thus predicting unprecedented universality of the KPZ class that controls even the distribution of the fluctuations yet depends on the global geometry of the interfaces. The experiment by Miettinen *et al.* [4] was the first attempt to test these theoretical predictions for flat interfaces and reached conclusions in favor of them as mentioned above.

Here, however, given their experimental accuracy, I argue that their analysis is not sufficient to make a significant distinction among these non-Gaussian distributions and thus is incapable of making a conclusion for or against the theoretical predictions. First of all, one should be aware that, though the GOE and GUE Tracy-Widom distributions as well as the F_0 distribution are clearly distinguishable in their original definition of the stochastic variable (inset of Fig. 1), the difference is quite subtle when they are arbitrarily shifted and scaled (Fig. 1) as was done by Miettinen *et al.* [4]. When they are normalized to have zero mean and unit variance, for example, the difference is barely visible only beneath approximately 10^{-4},

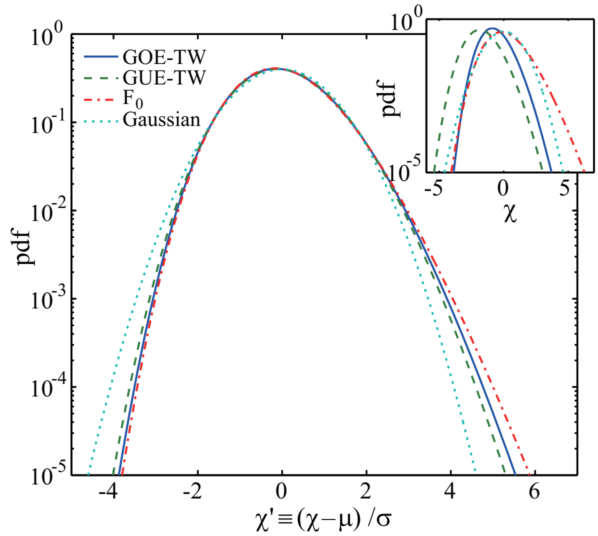

Fig. 1. Comparison of the GOE and GUE Tracy-Widom distributions (solid and dashed lines, respectively), the F_0 distribution (dashed-dotted line), and the Gaussian distribution with zero mean and unit variance (dotted line). Shown are the theoretical curves for their pdf (inset) and those normalized to have zero mean and unit variance (main panel). Note that the conventional definition of the random variable χ for the GOE Tracy-Widom distribution [5] is multiplied by $2^{-2/3}$ to conform with the theoretical prediction in Ref. [7].

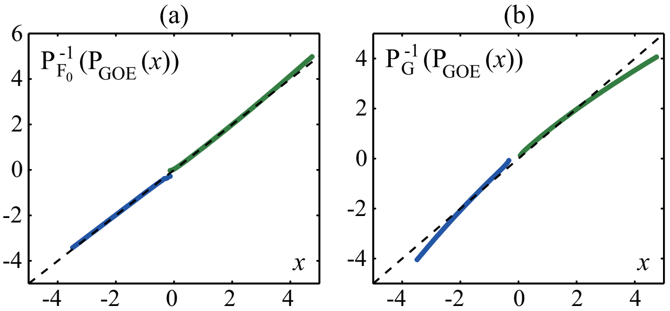

Fig. 2. Inversion plots for two given distributions: the GOE Tracy-Widom and F_0 distributions (a), and the GOE Tracy-Widom and Gaussian distributions (b). $P_{\mathrm{GOE}}(x)$, $P_{F_0}(x)$, and $P_{\mathrm{G}}(x)$ refer to the pdfs for the GOE Tracy-Widom, F_0, and Gaussian distributions, respectively, normalized with mean zero and unit variance as shown in the main panel of Fig. 1. The two sets of points (blue and green) correspond to the left and right branches of the function, respectively, and the dashed line indicates the identity transformation.

which is obscured by statistical and experimental errors in most of the data presented in Ref. [4].

Miettinen *et al.* [4] also compared and found in agreement the experimental and theoretical pdfs, $P_{\mathrm{exp}}(x)$ and $P_{\mathrm{theor}}(x)$, respectively, by showing $P_{\mathrm{exp}}^{-1}(P_{\mathrm{theor}}(x)) \approx x$ except for large x. These inversion plots are, however, not able to highlight the subtle differences in the pdfs either. Figure 2(a) reproduces their inversion plot, using the normalized pdfs for the GOE Tracy-Widom and F_0 distributions. It apparently indicates agreement, in particular for small x, but the two distributions are different ones, which Miettinen *et al.* attempted to distinguish. Shown in this way, actually, even the GOE Tracy-Widom and Gaussian distributions are not clearly distinguishable, if reliable data are obtained only for small x [Fig. 2(b)]. Although it seems unlikely that the distributions presented by Miettinen *et al.* are Gaussian, in particular because they reported positive values of the skewness [4], it would be more useful if they had provided ranges of error for their estimates of the skewness. As a matter of fact, the two reported values, 0.33 and 0.32, are both in between those for the GOE Tracy-Widom and F_0 distributions, 0.2935 and 0.359, respectively [4].

In order to distinguish the possibly different distributions in their experimental data, I propose rescaling the height fluctuations with measured parameter values, instead of fitting individually to theoretical curves, as performed in the recent experiment on growing interfaces of liquid-crystal turbulence [11,12]. This method relies on the fact that, given the growth exponent β is 1/3, the local height of the interface $h(x,t)$ is described as

$$h(x,t) \simeq v_\infty t + (\Gamma t)^{1/3}\chi \tag{1}$$

with constant parameters v_∞ and Γ and a random variable χ, which should be directly compared with the theoretical distributions. Then, one can estimate the two parameters and extract the random variable χ as follows:

1. Determine the asymptotic growth rate v_∞ by measuring $\mathrm{d}\langle h\rangle/\mathrm{d}t$ as a function of time. Since $\mathrm{d}\langle h\rangle/\mathrm{d}t \simeq v_\infty + at^{-2/3}$ with a constant a, one should plot $\mathrm{d}\langle h\rangle/\mathrm{d}t$ against $t^{-2/3}$ and read the y-intercept.
2. Determine the amplitude Γ of the interface fluctuations. This can be done most accurately by measuring the second-order cumulant $\langle h^2\rangle_{\mathrm{c}} \equiv \langle (h-\langle h\rangle)^2\rangle \simeq (\Gamma t)^{2/3}\langle\chi^2\rangle_{\mathrm{c}}$. Note that one needs to set here the value of $\langle\chi^2\rangle_{\mathrm{c}}$. This can be chosen arbitrary, e.g., $\langle\chi^2\rangle_{\mathrm{c}} = 1$, but one may choose the variance of the compared theoretical distribution to facilitate the comparison. This choice does not bias the results [11].
3. Extract the rescaled height χ using the measured parameter values and Eq. (1). Make a histogram to compare with the theoretical pdfs (not shifted and rescaled) and, more quantitatively, plot time series of the difference in the nth-order cumulants.

This method was indeed used in the liquid-crystal experiment and successfully distinguished the GOE and GUE Tracy-Widom distributions in the transient regime [11, 12]. It should also be suitable for the stationary regime when the height is appropriately defined and the two parameters are measured again.

As already stressed, the universal distributions of the KPZ class are in the midst of extensive theoretical investigations [8–10], but have been experimentally tested only in the paper combustion by Miettinen *et al.* [4] and in the liquid-crystal turbulence [11,12] yet. It is therefore of fundamental importance to carry out a quantitative test for the paper combustion experiment, as suggested in this Comment or otherwise. It is important in particular because it allows studying the stationary interfaces, which are not yet realized in the liquid-crystal turbulence, and also because Miettinen *et al.* reported the existence of "avalanches" due to quenched disorder [4], which are not present in all the known models of the KPZ class. Investigating other statistical properties known to exhibit the geometry-dependent universality, such as the spatial correlation, is also intriguing, as studied experimentally in Ref. [12].

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