Directed percolation criticality in turbulent liquid crystals

Kazumasa A. Takeuchi[1], Masafumi Kuroda[1], Hugues Chaté[2], and Masaki Sano[1]

[1]*Department of Physics, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo, 113-0033, Japan.*
[2]*Service de Physique de l'État Condensé, CEA-Saclay, 91191 Gif-sur-Yvette, France.*

Binary reduction of images

Every analysis presented in the paper is performed using binarized images, where DSM2 domains are distinguished from the absorbing DSM1 background. Although the principles of this binary reduction are already shown in the paper, we give here a detailed description of the algorithm we used.

The binarization is carried out in the following way: (a) We prepare three successive images taken at 15 frames per second, and remove the inhomogeneity of the incident light intensity. (b) We then normalize the obtained intensity I of the three images with respect to mean $\langle I_{\mathrm{DSM1}} \rangle$ and standard deviation δI_{DSM1} of the DSM1 intensity at a given voltage, namely $I_{\mathrm{norm}} = (I - \langle I_{\mathrm{DSM1}} \rangle)/\delta I_{\mathrm{DSM1}}$. Note that we can separately measure the intensity of the pure DSM1 state even above the threshold V_{c}, since DSM1 always appears first when the voltage is applied. (c) Since DSM2 domains appear darker than DSM1, we extract the regions where the normalized intensity is less than -1.5. (This threshold value is determined so as to obtain a good agreement with direct visual observations, notably in movies.) (d) Taking into consideration that DSM2 domains move much slower than the local intensity fluctuations in DSM1 (remember that DSM1 is itself a turbulent state) and that there is a minimum DSM2 area of $d^2/2$ [S1], where d is the depth of the cell, we take the logical intersection ("AND" operator) of the three successive images, and then keep only clusters whose area is larger than $d^2/2$. The clusters left in this step come from the middle image before the intersection is taken, in other words the intersection is used only for the comparison with the minimum area. (e) Finally, we cut off the periphery of the image of width $d/2\sqrt{2}$, since this region is biased in the step (d). The size of the binarized images reduces to $1206\,\mu\text{m} \times 899\,\mu\text{m}$, which roughly corresponds to 142×106 degrees of freedom.

All the images taken from our experiments are binarized in this way. We also confirmed that the single threshold value used (-1.5 in the normalized intensity) works well all over the range of voltages we investigate, and that no DSM2 region is falsely detected when we binarize images of the pure DSM1 phase. Typical results of the binarization are shown in Fig. 1c and Movie S2, where we can confirm that DSM2 domains are precisely detected.

Supplementary Table

Table S1: Comparison between experimentally measured exponents and those of DP [(1+1)-dimensional DP: above, (2+1)-dimensional DP: below]. RB is short for Rayleigh-Bénard convection. System sizes listed follow the indications given by the corresponding authors. Asterisks indicate that the value of the corresponding exponent is derived from other exponents. Estimates without confidence interval indicate that errors are not estimated in papers, except the values for the 1-dimensional DP, whose confidence intervals do not appear in the digits shown below. For our experiment, the system size with and without parentheses indicates the size of the observation area and of the convection area, respectively. The estimates of the exponents $\nu_\perp$ and $\mu_\perp$ in the x direction are shown in the first row and those in the y direction are in the second row.

(1+1)D System	Size	β	$\nu_\perp$	$\nu_\parallel$	$\mu_\perp$	$\mu_\parallel$	α	Refs.
annular RB	22		0.5		1.9(1)	1.9		[S2]
annular RB	35		0.5	0.5	1.7(1)	2.0(1)		[S3]
linear RB	25.7	0.30(5)	0.50(5)	0.50(5)	1.6(2)	2.0(2)		[S3]
interface roughening			$\nu_\perp/\nu_\parallel = 0.63(4)$					[S4]
viscous fingering	60	0.45(5)			0.64(2)	0.61(2)		[S5]
vortices of fluid	15	0.5				1.7		[S6]
Taylor-Deen	90	1.30(26)	0.64, 0.53	0.73	1.67(14)	1.74(16)		[S7]
Taylor-Couette	70	1	0.4		1.4-2.5			[S8]
granular flow			$\nu_\parallel - \nu_\perp = 1$					[S9,S10]
ferrofluidic spikes	108	0.30(5)	1.1(2)	0.62(14)	1.70(5)	2.1(1)		[S11]
DP		0.276	1.097	1.734	1.748^*	1.841^*	0.159^*	[S12]
		$\nu_\perp/\nu_\parallel = 0.633,$		$\nu_\parallel - \nu_\perp = 0.637$				

(2+1)D System	Size	β	$\nu_\perp$	$\nu_\parallel$	$\mu_\perp$	$\mu_\parallel$	α	Refs.
liquid columns	169	0.56(5)						[S13]
DSM1-DSM2	2.7×10^6	0.59(4)	$0.66(17)^*$	$1.18^{+(14)*}_{-(21)}$	1.10(22)	1.61(6)	$0.50^{+(8)}_{-(5)}$	
(present paper)	(1.5×10^4)		$0.77(7)^*$		1.23(4)			
DP		$0.583(3)^*$	$0.733(3)^*$	1.295(6)	$1.204(2)^*$	$1.5495(10)^*$	$0.4505(10)^*$	[S14,S15]

References

[S1] S. Kai, W. Zimmermann, M. Andoh, and N. Chizumi, J. Phys. Soc. Jpn. **58**, 3449 (1989).

[S2] S. Ciliberto and P. Bigazzi, Phys. Rev. Lett. **60**, 286 (1988).

[S3] F. Daviaud, M. Bonetti, and M. Dubois, Phys. Rev. A **42**, 3388 (1990).

[S4] S. V. Buldyrev *et al.*, Phys. Rev. A **45**, R8313 (1992).

[S5] S. Michalland, M. Rabaud, and Y. Couder, Europhys. Lett. **22**, 17 (1993).

[S6] H. Willaime, O. Cardoso, and P. Tabeling, Phys. Rev. E **48**, 288 (1993).

[S7] M. M. Degen, I. Mutabazi, and C. D. Andereck, Phys. Rev. E **53**, 3495 (1996).

[S8] P. W. Colovas and C. D. Andereck, Phys. Rev. E **55**, 2736 (1997).

[S9] A. Daerr and S. Douady, Nature **399**, 241 (1999).

[S10] H. Hinrichsen, A. Jiménez-Dalmaroni, Y. Rozov, and E. Domany, Phys. Rev. Lett. **83**, 4999 (1999).

[S11] P. Rupp, R. Richter, and I. Rehberg, Phys. Rev. E **67**, 036209 (2003).

[S12] I. Jensen, J. Phys. A: Math. Gen. **32**, 5233 (1999).

[S13] C. Pirat, A. Naso, J. -L. Meunier, P. Maïssa, and C. Mathis, Phys. Rev. Lett. **94**, 134502 (2005).

[S14] P. Grassberger and Y. C. Zhang, Physica A **224**, 169 (1996).

[S15] C. A. Voigt and R. M. Ziff, Phys. Rev. E **56**, R6241 (1997).