Partial yet definite emergence of the Kardar-Parisi-Zhang class in isotropic spin chains

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Integrable spin chains with a continuous non-Abelian symmetry, such as the one-dimensional isotropic Heisenberg model, show superdiffusive transport with little theoretical understanding. Although recent studies reported a surprising connection to the Kardar-Parisi-Zhang (KPZ) universality class in that case, this view was most recently questioned by discrepancies in full counting statistics. Here, by combining extensive numerical simulations of the Landau-Lifshitz one-dimensional magnet, with a framework developed by exact studies of the KPZ class, we characterize various two-point quantities that remain hitherto unexplored in spin chains, and find full agreement with KPZ scaling laws. This establishes the partial emergence of the KPZ class in isotropic spin chains. Moreover, we reveal that the KPZ scaling laws are intact in the presence of an energy current, under the appropriate Galilean boost required by the propagation of spacetime correlation.

Characterization of transport properties in quantum many-body systems, in particular those of integrable systems with non-diffusive transport, is one of the frontiers of condensed matter physics. Integrability typically results in ballistic transport, as successfully described by the framework of the generalized hydrodynamics [1–3], but it is faced with challenges when ballistic contributions are canceled by symmetry or other mechanisms [4, 5]. Paradigmatic is the situation with a continuous non-Abelian symmetry, in particular the isotropic Heisenberg spin chain, which was reported to show superdiffusive transport with the characteristic length $\xi(t) \sim t^{2/3}$ [6–8]. Surprisingly, this superdiffusive exponent was associated with an apparently unrelated universality class established mainly for classical non-equilibrium systems, namely the Kardar-Parisi-Zhang (KPZ) universality class for fluctuations of growing interfaces and related phenomena [9]. Key evidence [10] was the precise agreement of the equilibrium two-point spin correlation function with Prähofer and Spohn's exact solution for the KPZ class [11], often denoted by $f_{\mathrm{KPZ}}(\cdot)$. On the one hand, this alleged manifestation of the KPZ class is deemed universal [4, 5, 12], as confirmed in various isotropic integrable spin chains, whether quantum [13] or classical [14], and also supported by a few experimental investigations [15, 16]. On the other hand, it is clear from the symmetry of spins that the magnetization transfer (integrated spin current) must show a symmetric distribution unless the symmetry is explicitly broken by the initial condition or an external field [17], while for KPZ the corresponding quantity, namely the interface height increment, is intrinsically asymmetric [9]. Recent computational studies [18, 19] further backed this view, by showing clear evidence that even the kurtosis and other ratios of even-order cumulants, which are unaffected by the spin's up-down symmetry, are markedly different from the predictions of the KPZ class, calling for a new universality class to describe this class of systems. After all, all pieces of evidence for KPZ reported so far have been rather weak, being the scaling exponents, which are simple rational numbers such as 2/3, and the agreement with Prähofer and Spohn's solution $f_{\mathrm{KPZ}}(\cdot)$, which has been compared with arbitrarily fitted scaling coefficients.

Here we clarify the fate of the KPZ universality in isotropic integrable spin chains. First we remark that the deep body of knowledge gained by mathematical studies on the 1D KPZ class [9] has not been fully utilized. It dictates, for example, the mutual relation between scaling coefficients. They contain universal quantifiers, which are lost if treated as free fitting parameters. Moreover, the Prähofer-Spohn function is not the only two-point correlator with an exact solution [9]; other two-point functions, such as the equal-time spatial correlator [20] and equal-position two-time correlator [21, 22] have also been dealt with. The purpose of the present Letter is to make full use of these results to carry out a comprehensive test of the KPZ universality in isotropic integrated spin chains.

In the following, we mainly present results obtained for the isotropic version of the integrable lattice model introduced by Krajnik *et al.* [17, 23], based on the Landau-Lifshitz (LL) magnet [24]. The time evolution is given conceptually by

$$\frac{\partial \boldsymbol{S}_j}{\partial t} = \boldsymbol{S}_j \times (\boldsymbol{S}_{j+1} + \boldsymbol{S}_{j-1}), \tag{1}$$

but implemented in a specific manner on a brick-wall

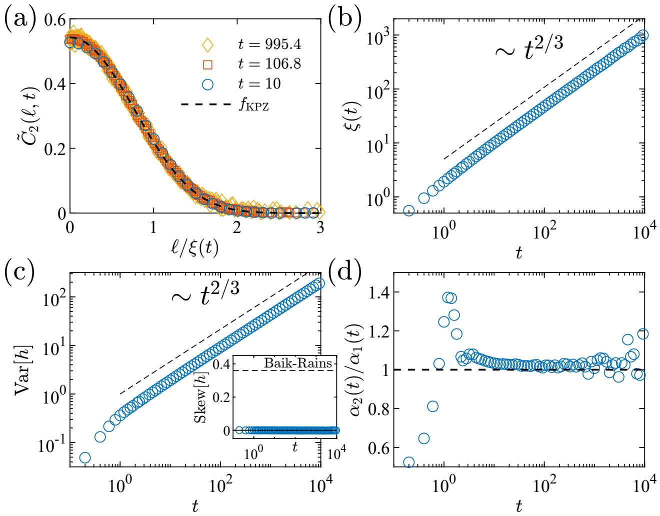

FIG. 1. The two-point function and the magnetization transfer cumulants for the LL model. (a) Rescaled two-point function $\tilde{C}_2(\ell,t) \equiv \frac{\xi(t)}{\Omega}C_2(\ell,t)$ against $\ell/\xi(t)$, compared with the Prähofer-Spohn exact solution $f_{\rm KPZ}(\cdot)$. (b) Correlation length $\xi(t)$. (c) Variance (main panel) and skewness (inset) of the magnetization transfer h. The dashed line in the inset indicates the skewness of the Baik-Rains distribution. (d) Ratio of the coefficients $\alpha_1(t)$ and $\alpha_2(t)$ evaluated from Eqs. (2) and (3), respectively.

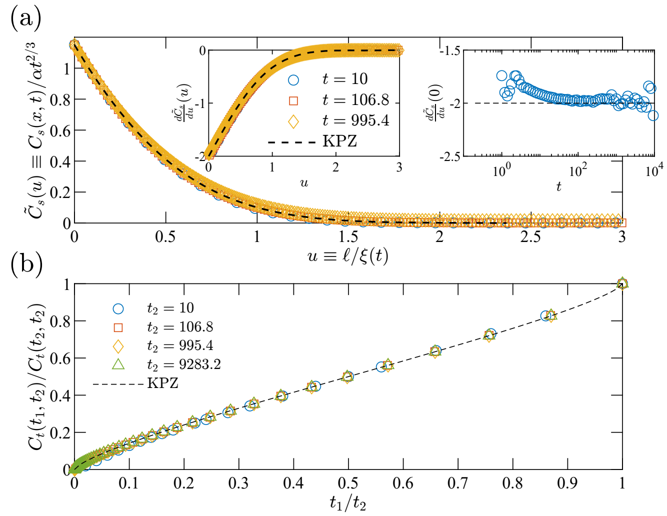

FIG. 2. The spatial (a) and temporal (b) correlation functions of the magnetization transfer for the LL model. (a) The rescaled spatial correlation function $\tilde{C}_s(u) = C_s(\ell,t)/\alpha t^{2/3}$ (main panel) and its slope $\frac{d\tilde{C}_s}{du}(u)$ (left inset) as functions of $u \equiv \ell/\xi(t)$. The dashed lines show the curves for the KPZ class, numerically obtained by TASEP simulations. The right inset compares the slope $\frac{d\tilde{C}_s}{du}(0)$ at $u=0$ with our exact result for the KPZ class, $\frac{d\tilde{C}_s}{du}(0) = -2$. (b) The rescaled temporal correlation function $C_t(t_1,t_2)/C_t(t_2,t_2)$ against t_1/t_2, compared with the Ferrari-Spohn exact solution [Eq. (6)] for the KPZ class.

space-time lattice [17, 23] to preserve the integrability of the LL magnet in continuous space-time. Here, unless otherwise stated, we started from infinite-temperature equilibrium states and obtained $N = 10^4$ independent realizations with system size $L = 40,000$ and the periodic boundary condition, with time step 0.1. The z-component of the spins, $S_j^z(t)$, is our magnetization field, denoted by $m(x,t)$ with $x = j$ hereafter. Another quantity of interest is the integrated spin current, or the magnetization transfer, $h(x,t) \equiv \int_0^t J(x,t')dt'$, with spin current $J(x,t)$. The magnetization transfer $h(x,t)$ corresponds to the height increment of the growing interfaces, which is central in the studies of the KPZ class.

First we verify KPZ behavior of the LL model through the standard quantities. Figure 1(a) displays the two-point function $C_2(\ell,t) \equiv \langle m(x+\ell,t)m(x,0)\rangle$, showing agreement with the Prähofer-Spohn exact solution $f_{\rm KPZ}(\cdot)$. Here the normalized function $\tilde{C}_2(\ell,t) \equiv \frac{\xi(t)}{\Omega}C_2(\ell,t)$ is shown, where $\Omega \equiv \int C_2(\ell,t)d\ell$ is conserved as a result of the conservation of the total magnetization $\int m(x,t)dx$, and $\xi(t)$ is the correlation length determined by $\frac{1}{\Omega}\int \ell^2 C_2(\ell,t)d\ell = \sigma^2\xi(t)^2$ with $\sigma^2 \equiv \int u^2 f_{\rm KPZ}(u)du \approx 0.51$. This correlation length is confirmed to show the characteristic power law $\xi(t) \sim t^{2/3}$ of the KPZ class [Fig. 1(b)]. We also measure the variance of the magnetization transfer and find the power-law growth with the KPZ characteristic exponent, $\mathrm{Var}[h(x,t)] \sim t^{2/3}$ [Fig. 1(c) main panel]. On the other hand, the skewness is zero and far from the value for the Baik-Rains distribution [25] expected for the KPZ stationary state (inset). Although the data shown so far are reproduction of known results [17, 18], we can scrutinize nontrivial relationship underlying these quantities. According to KPZ scaling laws [9, 11], we have

$$C_2(\ell,t) \simeq \frac{2\alpha t^{2/3}}{\xi(t)^2} f_{\rm KPZ}\left(\frac{\ell}{\xi(t)}\right), \tag{2}$$

$$\mathrm{Var}[h(x,t)] \simeq \alpha t^{2/3}\mathrm{Var}[\mathrm{BR}], \tag{3}$$

where $\mathrm{Var}[\mathrm{BR}] \approx 1.15$ is the variance of the Baik-Rains distribution and α is a coefficient. Since these equations are not guaranteed to describe spin chains, here we evaluate α from data of $C_2(x,t)$ and $\mathrm{Var}[h(x,t)]$ independently and denote them by $\alpha_1(t)$ and $\alpha_2(t)$, respectively. Then, remarkably, we find $\alpha_1(t) = \alpha_2(t)$ [Fig. 1(d)], substantiating the validity of the KPZ scaling laws (2) and (3) in spin chains. Therefore, interestingly, the Baik-Rains variance turns out to be in the formula (3), even though the Baik-Rains distribution itself does not appear.

We further test the validity of KPZ scaling laws in spin chains through other two-point quantities. First we study the spatial correlation of the magnetization transfer:

$$C_s(\ell,t) \equiv \langle h(x,t)h(x+\ell,t)\rangle - \langle h(x,t)\rangle^2. \tag{4}$$

Figure 2(a) shows it in the rescaled units, $\tilde{C}_s(u) \equiv C_s(\ell,t)/\alpha t^{2/3}$ against $u \equiv \ell/\xi(t)$. For the KPZ class, the multi-point equal-time height correlation has been characterized intensively and described in terms of a family of stochastic processes called the Airy processes [20, 26–30]. For the stationary state, a process called the $\mathrm{Airy}_{\mathrm{stat}}$ process has been considered [31] (see also a review [20]), but it describes the height measured in the absolute frame (say, $h_0(x,t)$) instead of the height increment $h(x,t) = h_0(x,t) - h_0(x,0)$ considered here. We therefore introduce here the limiting process $\mathcal{A}_0(u)$ for the height increment $h(x,t)$, in other words the stationary version of the $\mathrm{Airy}_{\mathrm{stat}}$ process, and call it the Airy_0 process. We evaluate the covariance of the Airy_0 process $\mathcal{C}_0(u) \equiv \langle \mathcal{A}_0(u)\mathcal{A}_0(0)\rangle$ by numerical simulations of the totally asymmetric simple exclusion process (TASEP), a paradigmatic model in the KPZ class, and find it in excellent agreement with the data for the LL model [Fig. 2(a)]. Furthermore, we consider $\mathcal{C}_0(u)$ for small u analytically and prove $\frac{d\mathcal{C}_0}{du}(0) = -2$ (see Supplementary Material), which is a characteristic distinct from the other known Airy processes for which the slope of the covariance at $u = 0$ is -1. This is confirmed by our numerical data for both the LL model and the TASEP [insets of Fig. 2(a)]. Finally, we also investigate the temporal correlation of the magnetization transfer

$$C_t(t_1,t_2) \equiv \langle h(x,t_1)h(x,t_2)\rangle - \langle h(x,t_1)\rangle\langle h(x,t_2)\rangle. \quad (5)$$

The results in Fig. 2(b) show excellent agreement with the exact solution for the KPZ class obtained by Ferrari and Spohn [21, 22]:

$$\frac{C_t(t_1,t_2)}{C_t(t_2,t_2)} \simeq \frac{1}{2}\left[1 + \tau^{2/3} - (1-\tau)^{2/3}\right], \quad (6)$$

with $\tau \equiv t_1/t_2$.

Now that we verified the validity of the KPZ scaling laws in various two-point quantities, we test its robustness under different situations. First, we consider the case with a non-vanishing energy current. This is particularly tempting in view of the hydrodynamic description proposed by De Nardis *et al.* [32], which predicts that left-moving and right-moving giant quasiparticles contribute equally to the magnetization, and this is why the distribution of h becomes symmetric. Therefore, it is important to clarify what happens if the left-right symmetry is broken, e.g., by the presence of a finite energy current. We prepared such an initial condition by Monte Carlo sampling, using the statistical weight $\propto e^{-\lambda J_E}$ with total energy current $J_E \equiv -\sum_j \boldsymbol{S}_j \cdot (\boldsymbol{S}_{j+1} \times \boldsymbol{S}_{j+2})$ [33] and $\lambda = 1$. Thereby, we indeed realize a situation where the energy current reaches a constant finite value after a short transient [Fig. 3(a) inset].

Figure 3(a) shows the two-point function $\tilde{C}_2(\ell,t)$ in this case. Interestingly, now we find the peak position of the correlation function moving at a constant velocity, $\ell_{\mathrm{peak}} = v_{\mathrm{peak}}t$ with $v_{\mathrm{peak}} = 0.5522$ [Fig. 3(b)(c)]. Apart from this, the form of the two-point function turns out to be unchanged, i.e., it is the Prähofer-Spohn function $f_{\mathrm{KPZ}}(\cdot)$ [Fig. 3(d)], and so does the growth of the correlation length, $\xi(t) \sim t^{2/3}$ [Fig. 3(e)]. Therefore, the KPZ physics remains intact in the presence of a finite energy current. The propagation of the space-time correlation revealed in Fig. 3(a)-(c) is analogous to the case of growing tilted interfaces [35, 36] and nonlinear fluctuating hydrodynamics for unharmonic chains [37]. An important lesson from these past studies is that one should measure the magnetization transfer $h(x,t)$ in the frame comov-

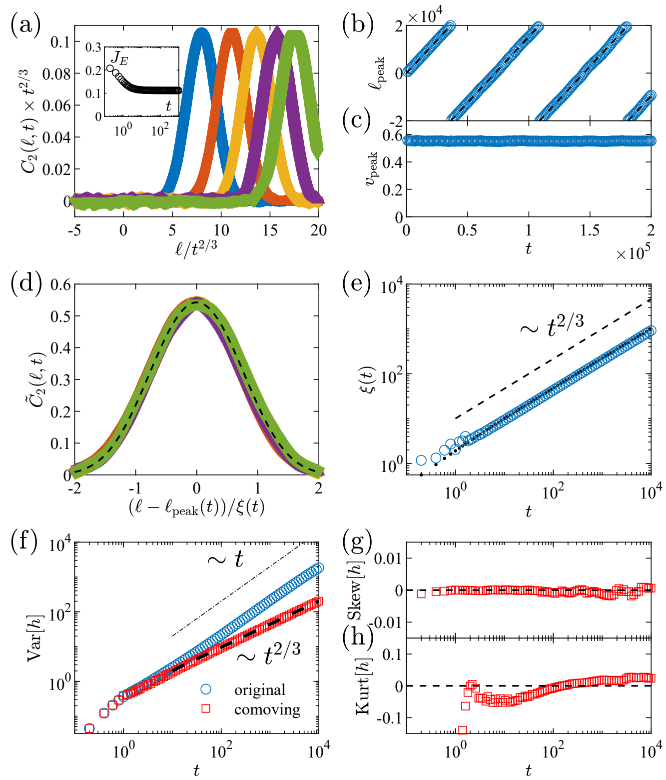

FIG. 3. Results for the case with a finite energy current. (a) Rescaled two-point function $\tilde{C}_2(\ell,t) \equiv \frac{\xi(t)}{\Omega}C_2(\ell,t)$ against $\ell/\xi(t)$ for different times, $t = 3000, 8000, 15000, 23000, 32000$ from left to right. The displayed data are smoothed and vertically shifted to have $\tilde{C}_2(\ell,t) = 0$ for ℓ far from the peak. Inset: total energy current $J_E(t)$. (b)(c) The location of the peak of $\tilde{C}_2(\ell,t)$, $\ell_{\mathrm{peak}}(t)$ (b), and its propagation speed v_{peak} (c). The dashed line in (b) shows $\ell_{\mathrm{peak}}(t) = vt$ with $v = 0.5522$, wrapped by the periodic boundary. (d) Rescaled two-point function centered at $\ell = \ell_{\mathrm{peak}}(t)$ (symbols, same colors as (a)), compared with the Prähofer-Spohn exact solution $f_{\mathrm{KPZ}}(\cdot)$ (dashed line). (e) Correlation length $\xi(t)$. The black dots are the data for the case without energy current, shown in Fig. 1(b) (f) Variance of the magnetization transfer $h(x,t)$, measured in the original and comoving frames (blue circles and red squares, respectively). The thick dashed line indicates the KPZ growth law Eq. (3) with the value of α determined from $C_2(\ell,t)$. (g)(h) Skewness (g) and kurtosis (h) of the magnetization transfer $h(x,t)$ in the comoving frame. The values for the Baik-Rains distribution are 0.359 and 0.289, respectively [34], which are far from the data.

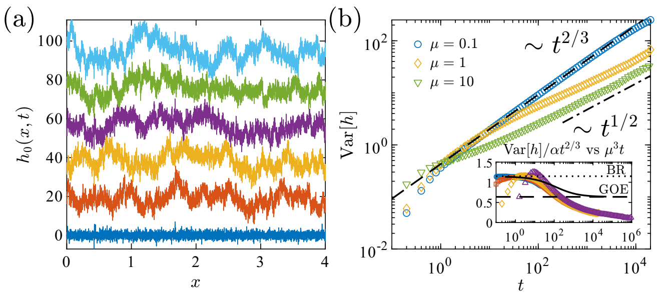

FIG. 4. Results for the flat initial condition. (a) Snapshots of the height $h_0(x,t) = h(x,t) + h_0(x,0)$ at $t = 0, 2000, 4000, \cdots, 10000$ from bottom to top, for $\mu = 1$. For visibility, every subsequent snapshot is shifted upward by 20. (b) Variance of the magnetization transfer h for different μ. The dashed line is the KPZ growth law (3) with α determined from the equilibrium simulations [Fig. 1(d)]. The dashed-dotted line is a guide for the eyes showing $\mathrm{Var}[h] \sim t^{1/2}$. Inset: rescaled variance $\mathrm{Var}[h]/\alpha t^{2/3}$ against $\mu^3 t$, for $\mu = 0.1, 0.5, 1, 2$. The bold solid line displays the behavior for KPZ interfaces [38] (with arbitrary horizontal shift), showing crossover from the Baik-Rains (BR) distribution (dotted line) to the characteristic distribution for flat interfaces, namely the GOE Tracy-Widom distribution (dashed line). Simulation parameters were $L = 40,000$ and $N = 1,000$ for $\mu = 0.1, 0.5, 1$ and $L = 400,000$ and $N = 33$ for $\mu = 2, 10$.

ing with the space-time correlator, which amounts to the following expression:

$$h(x,t) \equiv \int_0^t J(x,t')dt' - \int_{x-vt}^{x} m(x',0)dx' \qquad (7)$$

with $v = v_{\mathrm{peak}}$. With this appropriate definition of the magnetization transfer, we indeed confirm the KPZ growth of the variance, $\mathrm{Var}[h(x,t)] \sim t^{2/3}$ (more specifically, Eq. (3)) [Fig. 3(f) red squares], whereas the naïve definition $h(x,t) = \int_0^t J(x,t')dt'$ results in the apparent loss of the KPZ exponent (blue circles). On the other hand, even with the definition (7) without left-right symmetry, we do not find any indication of asymmetric distribution, as evidenced by vanishing skewness [Fig. 3(g)]. The value of the kurtosis also remains far from that of the Baik-Rains distribution [Fig. 3(h)], just like the case without energy current [18, 19]. To summarize, the presence of a finite energy current only necessitates considering the comoving frame; otherwise, it seems to have no effect on relevant statistical quantities, as long as they are measured in the comoving frame.

Finally, we study the effect of the initial condition. Universal statistical properties of the authentic KPZ class are known to depend on the initial condition, the three representative cases being the domain wall (curved interface), flat, and stationary initial conditions [9]. It is important to assess whether KPZ scaling laws for non-stationary cases can describe spin chains under the corresponding, non-equilibrium settings. Among these, the domain wall initial condition has been extensively studied

TABLE I. Two-point properties of KPZ confirmed in the equilibrium state of the LL magnet. The new results obtained in this work are marked with *.

exponents	$\xi(t) \sim t^{2/3}$, $\mathrm{Var}[h(x,t)] \sim t^{2/3}$
two-point function $C_2(\ell,t)$	Prähofer-Spohn solution f_{KPZ}
*variance amplitude	Eq. (3) with $\mathrm{Var}[\mathrm{BR}] \approx 1.15$
*spatial correlation $C_s(\ell,t)$	Airy_0 covariance, Fig. 2(a)
*time correlation $C_t(t_1,t_2)$	Ferrari-Spohn solution, Eq. (6)

(e.g., [6, 16]), and recent simulations suggested that KPZ may be observed only for finite times, being eventually replaced by the diffusive scaling [17]. This can be argued to result from the violation of the SU(2) symmetry, due to the chemical potential μ used to prepare each domain of biased spins [39]. Compared to this, the fate of the flat initial condition, i.e., the initial condition without bias of spins, is not clear and has not been studied to our knowledge, even if some recent simulations of quantum spin chains hint that KPZ behavior is also visible when starting from non-stationary states [40].

We realize a flat initial condition, by drawing each spin $\boldsymbol{S}_j(0)$ from infinite-temperature equilibrium distribution with a space-dependent vectorial chemical potential $\boldsymbol{\mu}_j$, $\rho(\boldsymbol{S}_j) = \frac{|\boldsymbol{\mu}_j|}{4\pi \sinh |\boldsymbol{\mu}_j|} e^{\boldsymbol{\mu}_j \cdot \boldsymbol{S}_j}$. The chemical potential is determined as follows: (i) $\boldsymbol{\mu}_1 = 0$, (ii) $\boldsymbol{\mu}_{j\geq 2} = -\mu \boldsymbol{S}_{j-1}^{\mathrm{tot}}/|\boldsymbol{S}_{j-1}^{\mathrm{tot}}|$ with $\boldsymbol{S}_{j-1}^{\mathrm{tot}} \equiv \sum_{j'=1}^{j-1} \boldsymbol{S}_{j'}$ and $\mu > 0$ [41]. This amounts to generating an initial height profile $h_0(j,0) \equiv \sum_{j'=1}^{j} S_{j'}^z$ that looks like a trajectory of an Ornstein-Uhlenbeck process [Fig. 4(a) bottom curve] instead of a Brownian trajectory for the equilibrium case $\mu = 0$. For KPZ interfaces, such initial conditions are expected to result asymptotically in the flat KPZ statistics [42], more precisely through a dynamical crossover from the stationary statistics (the Baik-Rains distribution) to the flat one (the GOE Tracy-Widom distribution) [38] without changing the scaling $\mathrm{Var}[h] \sim t^{2/3}$, as demonstrated for TASEP here (Fig. S1). In contrast, for the LL magnet, we find completely different behavior for $\mu > 0$, showing crossover from the KPZ scaling $t^{2/3}$ to the diffusive one $t^{1/2}$ [Fig. 4(b); see also Fig. 4(a) for the evolution of $h(x,t)$]. Close scrutiny of the behavior reveals that this crossover takes place at time scale μ^{-3} [Fig. 4(b) inset], in agreement with anomalous relaxation discussed in Ref. [39]. This indicates that the local violation of the isotropy (SU(2) for quantum spins) is sufficient for KPZ to break down in spin chains.

In summary, using the isotropic LL spin chain, we carried out quantitative tests of KPZ scaling laws for various two-point quantities that have not been characterized for spin chains so far, and found precise agreement in all of them (Table I). Nevertheless, the KPZ scaling laws seem to not describe higher-order quantities, as evidenced by earlier studies [18, 19]. Therefore, the strict KPZ class rules only a subset of statistical properties of isotropic integrable spin chains (and other cases with a continuous non-Abelian symmetry [12]). It is of primary importance to clarify the underlying principles of such partial emergence of the KPZ class. It could be explained by the coupled Burgers equation as proposed in Ref. [32] or by introducing a larger number of hydrodynamic modes in the system. Also one cannot exclude that there exists a hitherto unknown observable that can capture full KPZ statistics. Our finding on the robustness of the KPZ scaling in the presence of energy current, as well as its breakdown by the local violation of isotropy, may also be hints for probing this mystery, which hangs over such simple quantum many-body systems as the isotropic Heisenberg spin chain.

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