Partial Yet Definite Emergence of the Kardar-Parisi-Zhang Class in Isotropic Spin Chains

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Integrable spin chains with a continuous non-Abelian symmetry, such as the one-dimensional isotropic Heisenberg model, show superdiffusive transport with little theoretical understanding. Although recent studies reported a surprising connection to the Kardar-Parisi-Zhang (KPZ) universality class in that case, this view was most recently questioned by discrepancies in full counting statistics. Here, by combining extensive numerical simulations of classical and quantum integrable isotropic spin chains with a framework developed by exact studies of the KPZ class, we characterize various two-point quantities that remain hitherto unexplored in spin chains, and find full agreement with KPZ scaling laws without adjustable parameters. This establishes the partial emergence of the KPZ class in integrable isotropic spin chains. Moreover, we reveal that the KPZ scaling laws are intact in the presence of an energy current, under the appropriate Galilean boost required by the propagation of spacetime correlation.

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Characterizing transport properties of quantum manybody systems, in particular, those of integrable systems with nondiffusive transport, is a long-standing objective of condensed matter physics. Integrability typically results in ballistic transport, as successfully described by the framework of the generalized hydrodynamics [1-3], but it is faced with challenges when ballistic contributions are canceled by symmetry or other mechanisms [4,5]. Paradigmatic is the situation with a continuous non-Abelian symmetry, in particular, the isotropic Heisenberg spin chain, which was reported to show superdiffusive transport with characteristic length $\xi(t) \sim$ $t^{2/3}$ [6–9]. Surprisingly, this superdiffusive exponent was associated with an apparently unrelated universality class established mainly for classical nonequilibrium systems, namely, the Kardar-Parisi-Zhang (KPZ) universality class for fluctuations of growing interfaces and related phenomena [10]. Key evidence [11,12] was the precise agreement of the equilibrium two-point spin correlation function with Prähofer and Spohn's exact solution for the KPZ class [13], often denoted by $f_{\rm KPZ}(\cdot)$. On the one hand, this alleged manifestation of the KPZ class is deemed universal [4,5,14], as confirmed in various isotropic integrable spin chains, whether quantum [15] or classical [16], and also supported by a few experimental investigations [17,18]. On the other hand, it is clear from the symmetry of spins that the magnetization transfer (integrated spin current) must show a symmetric distribution, unless the symmetry is explicitly broken by the initial condition or an external field [19], while for KPZ the corresponding quantity, namely, the interface height increment, is intrinsically asymmetric [10]. Furthermore, recent computational studies [20,21] revealed discrepancies beyond the symmetry difference, notably in the kurtosis. These findings led the authors to set aside the possibility that spin transport in such systems may be described by the KPZ class, instead suggesting the need for a new universality class [20,21]. After all, all pieces of evidence for KPZ reported so far have been rather weak, being the scaling exponents, which are simple rational numbers such as 2/3, and the agreement with Prähofer and Spohn's solution $f_{KPZ}(\cdot)$, which has been compared with arbitrarily fitted scaling coefficients. Serious doubt is cast on the relevance of the KPZ class in this context, or more broadly in characterizing transport properties of a class of quantum many-body systems.

Here we clarify the fate of the KPZ universality in isotropic integrable spin chains, both classical and quantum.

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First we remark that the deep body of knowledge gained by mathematical studies on the 1D KPZ class [10] has not been fully utilized. It dictates, for example, the mutual relation between scaling coefficients. They contain universal quantifiers, which are lost if treated as free fitting parameters. Moreover, the Prähofer-Spohn function is not the only twopoint correlator with an exact solution [10]; other two-point functions, such as the equal-time spatial correlator [22] and equal-position two-time correlator [23,24] have also been dealt with. The purpose of the present Letter is to make full use of these results to carry out a comprehensive test of the KPZ universality in isotropic integrable spin chains. Our results on various two-point functions hitherto unexplored in spin chains reveal that the KPZ class is indeed relevant in spin chains, yet it describes two-point functions only, hence the partial emergence of the KPZ class.

We study both classical and quantum integrable isotropic spin chains. For the classical case, we use Krajnik and Prosen's model [19,25,26], based on the lattice Landau-Lifshitz magnet [27], which we shall call the KPLL model. It is an integrable variant of the lattice Landau-Lifshitz model defined on a brick-layer space-time lattice (see Supplemental Material text 1 and Fig. S1 [28] for the complete definition), which converges to the Ishimori chain [36]

$$\frac{\partial S_j}{\partial t} = \frac{S_j \times S_{j-1}}{1 + S_j \cdot S_{j-1}} + \frac{S_j \times S_{j+1}}{1 + S_j \cdot S_{j+1}},\tag{1}$$

in the continuous time limit [25,26]. For the quantum case, we study the isotropic Heisenberg chain, which is a representative integrable model [37] defined by

$$\hat{H} = \sum_{j} \hat{S}_{j} \cdot \hat{S}_{j+1}, \qquad (2)$$

with spin-1/2 operator $\hat{S}_i = (\hat{S}_i^x, \hat{S}_i^y, \hat{S}_i^z)$. In the following, we use the classical KPLL model to realize large-scale simulations for inspecting supposedly universal statistical properties of isotropic integrable spin chains, which are then confirmed by the quantum Heisenberg simulations. For the KPLL model, unless otherwise stated, we started from infinite-temperature equilibrium states and obtained $N = 10^4$ independent realizations with system size $L = 40\,000$ and the periodic boundary condition, with time step 0.1. The z component of the spins, $S_i^z(t)$, is our magnetization field, denoted by m(x, t) with x = j hereafter whenever appropriate. Another quantity of interest is the integrated spin current, or the magnetization transfer, $h(x, t) \equiv \int_0^t J(x, t') dt'$, with spin current J(x, t). The magnetization transfer h(x, t) corresponds to the height increment of the growing interfaces, which is central in the studies of the KPZ class.

First we verify KPZ behavior of the KPLL model through the standard quantities. Figure 1(a) displays the



FIG. 1. The two-point function and the magnetization transfer cumulants for the KPLL model. (a) Rescaled two-point function $\tilde{C}_2(\ell, t) = [\xi(t)/\Omega]C_2(\ell, t)$ against $\ell/\xi(t)$, compared with the Prähofer-Spohn solution $f_{\text{KPZ}}(\cdot)$. (b) Correlation length $\xi(t)$. (c) Variance (main panel) and skewness (inset) of the magnetization transfer *h*. The dashed line in the inset indicates the skewness of the Baik-Rains distribution. (d) Ratio of $\alpha_1(t)$ [from Eq. (3)] and $\alpha_2(t)$ [from Eq. (4)].

 $C_2(\ell, t) \equiv \langle m(x+\ell, t)m(x, 0) \rangle,$ two-point function showing agreement with the Prähofer-Spohn exact solution $f_{\text{KPZ}}(\cdot)$. Here the normalized function $\tilde{C}_2(\ell, t) \equiv$ $[\xi(t)/\Omega]C_2(\ell, t)$ is shown, where $\Omega \equiv \int C_2(\ell, t)d\ell$ is conserved as a result of the conservation of the total magnetization $\int m(x,t)dx$, and $\xi(t)$ is the correlation length determined by $(1/\Omega) \int \ell^2 C_2(\ell, t) d\ell = \sigma^2 \xi(t)^2$ with $\sigma^2 \equiv \int u^2 f_{\text{KPZ}}(u) du \approx 0.51$. This correlation length is confirmed to show the characteristic power law $\xi(t) \sim$ $t^{2/3}$ of the KPZ class [Fig. 1(b)]. We also measure the variance of the magnetization transfer and find the characteristic KPZ growth, $Var[h(x, t)] \sim t^{2/3}$ [Fig. 1(c)]. On the other hand, the skewness is zero and far from the value for the Baik-Rains distribution [38] expected for the KPZ stationary state (inset). Although the data shown so far are a reproduction of known results [19,20], we can scrutinize nontrivial relationship underlying these quantities. According to KPZ scaling laws [10,13,39], we have

$$C_2(\ell, t) \simeq \frac{2\alpha t^{2/3}}{\xi(t)^2} f_{\text{KPZ}}\left(\frac{\ell}{\xi(t)}\right),\tag{3}$$

$$\operatorname{Var}[h(x,t)] \simeq \alpha t^{2/3} \operatorname{Var}[\mathrm{BR}], \tag{4}$$

where Var[BR] ≈ 1.15 is the variance of the Baik-Rains distribution and α is a coefficient. Since these equations are not guaranteed to describe spin chains, here we evaluate α from data of $C_2(x, t)$ and Var[h(x, t)] independently and denote them by $\alpha_1(t)$ and $\alpha_2(t)$, respectively. Then,



FIG. 2. Spatial (a) and temporal (b) correlation functions of the magnetization transfer for the KPLL model. (a) The rescaled spatial correlator $\tilde{C}_s(u) = C_s(\ell, t)/\alpha t^{2/3}$ (main panel) and its slope $d\tilde{C}_s/du$ (left inset) against $u = \ell/\xi(t)$. The dashed lines show the curves for the KPZ class, obtained by TASEP simulations. The right inset compares the slope $d\tilde{C}_s/du$ at u = 0 with our exact result for the KPZ class, $d\tilde{C}_s/du = -2$. (b) The rescaled temporal correlator $C_t(t_1, t_2)/C_t(t_2, t_2)$ against t_1/t_2 , compared with the Ferrari-Spohn solution [Eq. (7)] for the KPZ class.

remarkably, we find $\alpha_1(t) = \alpha_2(t)$ [Fig. 1(d)], substantiating the validity of the KPZ scaling laws (3) and (4) in spin chains. Note that Eq. (4) includes the Baik-Rains variance, even though the Baik-Rains distribution does not appear in spin chains.

We further test the validity of KPZ scaling laws through other two-point quantities. First we study the spatial correlation of the magnetization transfer:

$$C_s(\ell, t) \equiv \langle h(x, t)h(x+\ell, t)\rangle - \langle h(x, t)\rangle^2.$$
(5)

Figure 2(a) shows it in the rescaled units, $\tilde{C}_s(u) \equiv$ $C_{\rm s}(\ell,t)/\alpha t^{2/3}$ against $u \equiv \ell/\xi(t)$. For the KPZ class, the multi-point equal-time height correlation has been characterized intensively and described in terms of a family of stochastic processes called the Airy processes [22,40-44]. For the stationary state, a process called the Airy_{stat} process has been considered [45] (see also a review [22]), but it describes the height measured in the absolute frame (say, $h_0(x, t)$ instead of the height increment $h(x, t) = h_0(x, t) - h_0(x, t)$ $h_0(x,0)$ considered here. We therefore introduce here the limiting process $\mathcal{A}_0(u)$ for the height increment h(x, t), in other words the stationary version of the Airy_{stat} process, and call it the Airy₀ process. We evaluate the covariance of the Airy₀ process $C_0(u) \equiv \langle A_0(u) A_0(0) \rangle$ by numerical simulations of the totally asymmetric simple exclusion process (TASEP), a representative model in the KPZ class, and find it in excellent agreement with the data for the KPLL model [Fig. 2(a)]. Furthermore, we consider $C_0(u)$



FIG. 3. Results for the quantum Heisenberg model, with system size L = 100 and maximum bond dimension $\chi = 1600$ (see End Matter for details). (a) Correlation length $\xi(t)$ and variance of the magnetization transfer, Var[h]. (b) Ratio $\alpha_2(t)/\alpha_1(t)$. See End Matter for the evaluation of the error bars. (c),(d) Spatial (c) and temporal (d) correlation functions of the magnetization transfer.

for small *u* analytically and prove $dC_0/du = -2$ at u = 0 (see Supplemental Material text 2 [28]). This is confirmed by our data for both the KPLL model and the TASEP [insets of Fig. 2(a)]. Finally, we also investigate the temporal correlation of the magnetization transfer

$$C_t(t_1, t_2) \equiv \langle h(x, t_1)h(x, t_2) \rangle - \langle h(x, t_1) \rangle \langle h(x, t_2) \rangle.$$
(6)

The results in Fig. 2(b) show excellent agreement with the exact solution for the KPZ class obtained by Ferrari and Spohn [23,24]:

$$\frac{C_t(t_1, t_2)}{C_t(t_2, t_2)} \simeq \frac{1}{2} \left[1 + \tau^{2/3} - (1 - \tau)^{2/3} \right],\tag{7}$$

with $\tau \equiv t_1/t_2$. Theoretically, Eq. (7) is for $t_1, t_2 \rightarrow \infty$ with a fixed τ , but the data converge remarkably fast.

The results obtained so far for the classical KPLL model verified the validity of the KPZ scaling laws in various twopoint quantities. Remarkably, we confirm all these results in simulations of the quantum Heisenberg model too (Fig. 3) without any adjustable parameter. This establishes the universality of the results, encompassing both the quantum and classical worlds. The details of the simulation and the results of the quantum model are presented in End Matter.

Now we test the robustness of our findings under different situations, again using the classical KPLL model. First, we consider the case with a non-vanishing energy current. This is particularly tempting in view of the hydrodynamic description proposed by De Nardis *et al.* [46], which predicts that left-moving and right-moving giant quasiparticles contribute equally to the magnetization,



FIG. 4. Results for KPLL with a finite energy current. (a) Twopoint function $C_2(\ell, t)/t^{2/3}$ against $\ell/\xi(t)$ for different times, t = 3000, 8000, 15000, 23000, 32000 from left to right. Data smoothed by the locally weighted scatterplot smoothing method are displayed. Inset: total energy current $J_E(t)$. (b),(c) The location and the velocity of the peak of $C_2(\ell, t)$, $\ell_{\text{peak}}(t)$ and v_{peak} , respectively. The dashed line in (b) shows $\ell_{\text{peak}}(t) = vt$ with v = 0.5523, wrapped by the periodic boundary. (d) Rescaled two-point function $\tilde{C}_2(\ell, t)$ centered at $\ell = \ell_{\text{peak}}(t)$ [symbols, same colors as (a)], compared with the Prähofer-Spohn solution $f_{\text{KPZ}}(\cdot)$ (dashed line). (e) Correlation length $\xi(t)$. The black dots are the data for the case without energy current, shown in Fig. 1(b). (f) Variance of the magnetization transfer h(x, t), measured in the original and comoving frames (blue circles and red squares, respectively). (g),(h) Skewness and kurtosis of the magnetization transfer h(x, t) in the comoving frame. The values for the Baik-Rains distribution are 0.359 and 0.289, respectively [49], which are far from the data.

and this is why the distribution of *h* becomes symmetric. Therefore, it is important to clarify what happens if the left-right symmetry is broken, e.g., by the presence of a finite energy current. We prepared such an initial condition by Monte Carlo sampling, using the statistical weight $\propto e^{-\lambda J_E}$ with total energy current $J_E \equiv -\sum_j S_j \cdot (S_{j+1} \times S_{j+2})$ [47] and $\lambda = -1$. Thereby, we indeed realize a situation where the energy current reaches a constant finite value after a short transient [Fig. 4(a) inset].

Figure 4(a) shows the two-point function $\tilde{C}_2(\ell, t) = [\xi(t)/\Omega]C_2(\ell, t)$ in this case. Interestingly, now we find the peak position of the correlation function moving at a constant velocity, $\ell_{\text{peak}} = v_{\text{peak}}t$ with $v_{\text{peak}} = 0.5523$

[Figs. 4(b) and 4(c)]. Apart from this, the form of the two-point function turns out to be unchanged, i.e., it is the Prähofer-Spohn function $f_{\text{KPZ}}(\cdot)$ [Fig. 4(d)] with correlation length growing as $\xi(t) \sim t^{2/3}$ [Fig. 4(e)]. Therefore, the KPZ physics remains intact in the presence of a finite energy current. The propagation of the space-time correlation revealed in Figs. 4(a)–4(c) is analogous to the case of growing tilted interfaces [50,51] and nonlinear fluctuating hydrodynamics for unharmonic chains [52]. An important lesson from these studies is that one should measure the magnetization transfer h(x, t) in the frame comoving with the space-time correlator, which amounts to the following expression:

$$h(x,t) \equiv \int_0^t J(x,t')dt' - \int_{x-vt}^x m(x',0)dx', \qquad (8)$$

with $v = v_{\text{peak}}$. With this appropriate definition of the magnetization transfer, we indeed confirm the KPZ growth of the variance, Eq. (4) [Fig. 4(f) red squares], whereas the naïve definition $h(x, t) = \int_0^t J(x, t') dt'$ fails to capture the KPZ exponent (blue circles). On the other hand, even with the definition (8) without left-right symmetry, we do not find any indication of asymmetric distribution, as evidenced by vanishing skewness [Fig. 4(g)]. The value of the kurtosis also remains far from that of the Baik-Rains distribution [Fig. 4(h)], just like the case without energy current [20,21]. To summarize, the presence of a finite energy current only necessitates considering the comoving frame; otherwise, it seems to have no effect on relevant statistical quantities, as long as they are measured in the comoving frame.

Finally, we study the effect of the initial condition. Universal statistical properties of the authentic KPZ class are known to depend on the initial condition, the three representative cases being the domain wall (curved interface), flat, and stationary initial conditions [10]. It is important to assess whether KPZ scaling laws for nonstationary cases can describe spin chains under the corresponding, nonequilibrium settings. For the domain wall initial condition, recent simulations suggested that KPZ may be observed only for finite times, being eventually replaced by the diffusive scaling [19]. This is argued to result from the violation of the SU(2) symmetry, due to the chemical potential μ used to prepare each domain of biased spins [53]. Compared to this, the fate of the flat initial condition is not clear and has not been studied to our knowledge, even if some recent simulations of quantum spin chains hint that KPZ behavior is also visible when starting from nonstationary states [54].

We realize a flat initial condition, by drawing each spin $S_j(0)$ from infinite-temperature equilibrium distribution with a space-dependent vectorial chemical potential μ_j , $\rho(S_j) = (|\mu_j|/4\pi \sinh |\mu_j|)e^{\mu_j \cdot S_j}$. The chemical potential is determined as follows: (i) $\mu_1 = 0$, (ii) $\mu_{j\geq 2} = -\mu S_{j-1}^{\text{tot}} / |S_{j-1}^{\text{tot}}|$



FIG. 5. Results for KPLL with the flat initial condition. (a) Snapshots of the height $h_0(x, t) = h(x, t) + h_0(x, 0)$ at t = 0, 2000, 4000, ..., 10 000 from bottom to top, for $\mu = 1$. For visibility, every subsequent snapshot is shifted upward by 20. (b) Variance of the magnetization transfer h for different μ . The dashed line is the KPZ growth law (4) with α determined from the equilibrium simulations [Fig. 1(d)]. The dashed-dotted line is a guide for the eyes showing $Var[h] \sim t^{1/2}$. Inset: rescaled variance $Var[h]/\alpha t^{2/3}$ against $\mu^3 t$, for $\mu = 0.1, 0.5, 1, 2$, from top to bottom at the leftmost part of the datasets. The bold solid line displays the behavior for KPZ interfaces [57] (with arbitrary horizontal shift), showing crossover from the Baik-Rains (BR) distribution (dotted line) to the characteristic distribution for flat interfaces, namely the GOE Tracy-Widom distribution (dashed line). Simulation parameters were $L = 40\,000$ and N = 1000 for $\mu = 0.1, 0.5, 1$ and $L = 400\,000$ and N = 33 for $\mu = 2, 10$.

with $S_{j-1}^{\text{tot}} \equiv \sum_{j'=1}^{j-1} S_{j'}(0)$, and $\mu > 0$ [55]. This amounts to generating an initial height profile $h_0(x, 0) \equiv$ $-\sum_{0 \le i \le x} S_i^z(0)$ that looks like a trajectory of an Ornstein-Uhlenbeck process [Fig. 5(a) bottom curve] instead of a Brownian trajectory for the equilibrium case $\mu = 0$. For KPZ interfaces, we demonstrate with TASEP that such initial conditions result asymptotically in the flat KPZ statistics [56], through a dynamical crossover from the stationary statistics (the Baik-Rains distribution) to the flat one (the GOE Tracy-Widom distribution) [57] without changing the scaling $Var[h] \sim t^{2/3}$ (Fig. S2 [28]). In contrast, for the KPLL magnet, we find completely different behavior for $\mu > 0$, showing crossover from the KPZ scaling $t^{2/3}$ to the diffusive one $t^{1/2}$ [Fig. 5(b)]. Close scrutiny reveals that this crossover takes place at time scale μ^{-3} [Fig. 5(b) inset], in agreement with anomalous relaxation discussed in Ref. [53]. This indicates that the local violation of the isotropy [SU(2) for quantum spins] is sufficient for KPZ to break down in spin chains.

In summary, using the integrable isotropic spin chains, both classical and quantum, we carried out quantitative tests of KPZ scaling laws for various two-point quantities that have not been characterized for spin chains so far, and found precise agreement in all of them (Table I). Nevertheless, the KPZ scaling laws seem to not describe higher-order quantities, as evidenced by earlier studies [20,21]. Therefore, as the main conclusion of the Letter, the strict KPZ class rules only a subset of statistical properties

TABLE I. KPZ two-point properties in equilibrium integrable spin chains. The new results of this work are marked with *.

Exponents	$\xi(t) \sim t^{2/3}, \operatorname{Var}[h(x, t)] \sim t^{2/3}$
Two-point function $C_2(\ell, t)$	Prähofer-Spohn solution f_{KPZ}
*Variance amplitude	Eq. (4) with $Var[BR] \approx 1.15$
*Spatial correlation $C_s(\ell, t)$	Airy ₀ covariance, Fig. $2(a)$
*Time correlation $C_t(t_1, t_2)$	Ferrari-Spohn solution, Eq. (7)

of isotropic integrable spin chains (and other cases with a continuous non-Abelian symmetry [14]). It is of primary importance to clarify the underlying principles of such partial emergence of the KPZ class. The coupled Burgers equations in Ref. [46] showed how KPZ two-point quantities can emerge out of symmetric one-point distributions, but the kurtosis remains unexplained. It is an open question if one can extend the theory to make it fully consistent, e.g., by introducing a larger number of hydrodynamic modes. Our finding on the robustness of the KPZ scaling in the presence of energy current, as well as its breakdown by the local violation of isotropy, may also be hints for probing this mystery, which hangs over such simple quantum many-body systems as the isotropic Heisenberg spin chain.

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End Matter

Appendix A: Quantum model simulation-To obtain the data shown in Fig. 3, we numerically evaluate $C_2(\ell, t), \quad C_s(\ell, t) \quad \text{and} \quad C_t(\ell, t) \quad \text{for the infinite}$ temperature equilibrium state. These quantities are defined for the quantum model as follows: $C_2(\ell, t) \equiv$ $C_{s}(\ell, t) \equiv \langle \Delta \hat{S}_{0}(t) \Delta \hat{S}_{\ell}(t) \rangle - \langle \Delta \hat{S}_{\ell}(t) \rangle^{2},$ $\langle \hat{S}_0^z \hat{S}_{\mathscr{L}}^z(t) \rangle$, and $C_t(t_1, t_2) \equiv \langle \Delta \hat{S}_0(t_1) \Delta \hat{S}_0(t_2) \rangle - \langle \Delta \hat{S}_0(t_1) \rangle \langle \Delta \hat{S}_0(t_2) \rangle$ where $\langle \hat{\mathcal{O}} \rangle \equiv \text{Tr}[\hat{\mathcal{O}}]/2^L$, $\hat{S}_i^{\alpha}(t) \equiv \hat{U}_t \hat{S}_j^{\alpha} \hat{U}_t^{\dagger}(\alpha = x, y, z)$ with $\hat{U}_t \equiv e^{-i\hat{H}t}$, $\hat{J}_i(t) \equiv \hat{S}_i^x(t)\hat{S}_{i+1}^y(t) - \hat{S}_i^y(t)\hat{S}_{i+1}^x(t)$ and $\Delta \hat{S}_i(t) \equiv \int_0^t \hat{J}_i(t') dt'$ represents the local spin current and magnetization transfer at the *i*th site, respectively. Here, the site index 0 represents the (L/2)th site counted from the left boundary and is nearly at the center of the system, where L is the total number of sites. The index j runs from -(L/2) + 1 to L/2. In the quantum context, the definition of charge transfer must come with a prescription for the appropriate time ordering of the operator $\Delta \hat{S}_i(t)$ as the current operators at different times do not commute with each other; see Refs. [58,59]. However, for simplicity, our correlation functions $C_s(\ell, t)$ and $C_t(t_1, t_2)$ are not defined with such a time ordering, even though these quantities are still physically observable in principle, as we argue in Supplemental Material text 3 [28]. Testing the KPZ scaling for correlation functions with the appropriate time ordering to reproduce charge transfer is an important problem left for future studies.

To obtain the time-evolved operators and their expectation values, we use the time-evolving block decimation (TEBD) method [60] for matrix product operators, with time step $\Delta t = 0.05$ and maximum bond dimension $\chi = 400, 800, 1200, 1600$, implemented by ITensor [61]. We consider L = 100 and the open boundary condition for this simulation. For $C_s(\ell, t)$ and $C_t(t_1, t_2)$, we first obtain the current-current correlation function $F_{\ell}(t) \equiv$ $\langle \hat{J}_{\ell}(0) \hat{U}_t \hat{J}_0(0) \hat{U}_t^{\dagger} \rangle$ using the TEBD and then compute numerical integrals, such as $\langle \Delta \hat{S}_0(t) \Delta \hat{S}_{\ell}(t) \rangle =$ $\int_0^t dt' \int_0^t dt'' F_{\ell}(t' - t'')$. The time integral is evaluated with a large time step $20\Delta t$.

For simulations of such quantum models, it is crucial to deal with truncation error due to the finite bond dimension



FIG. 6. Detailed results for the quantum Heisenberg model (system size L = 100 and open boundary condition). All panels except the inset of (c) show the χ dependence of $\Omega(t)$ (a), $\xi(t)$ (b), $\tilde{C}_2(\ell, t)$ (c), $\alpha_1^{\min}(t)$ and $\alpha_1^{\max}(t)$ (d), $\alpha_2(t)$ (e), $\operatorname{Var}[h]$ [(e) inset], $\alpha_2(t)/\alpha_1(t)$ (f), $C_s(\ell, t)$ (g), and $C_t(t_1, t_2)$ (h), where χ denotes the maximum bond dimension in the TEBD simulation. The same symbols are used in these panels as indicated in the legend of (a). The error bars in (f) are given by the range $[\alpha_2(t)/\alpha_1^{\max}(t), \alpha_2(t)/\alpha_1^{\min}(t)]$. The dashed lines in (c),(g),(h) are the Prähofer-Spohn solution $f_{\text{KPZ}}(u)$, the Airy₀ covariance $C_0(u)$, and the Ferrari-Spohn solution (7), respectively. The inset of (c) shows the time dependence of $\tilde{\alpha}_1(\ell, t)$ with $\chi = 1600$.

 χ . To understand the effect of this error, we have performed simulations with different χ and compared them. The results described in the following indicate that a large value of χ is needed for quantitative verification of the KPZ scaling functions, larger than that needed for the exponents. While most of the data shown in Fig. 3 are well converged with respect to χ within numerical accuracy, $\alpha_1(t)$ is not, and we have systematically evaluated the error bars as explained below. Note that the Trotter error is not the dominant source of error in the present study. Appendix B: Numerical results for the quantum model—Let us start with the quantities related to $C_2(\ell, t)$. Figure 6(a) shows $\Omega(t) \equiv \sum_l C_2(\ell, t)$, which should be conserved by the U(1) symmetry of the Hamiltonian [Eq. (2)]. However, because of the truncation error due to finite bond dimension, $\Omega(t)$ starts to deviate at long times. Indeed, we find that the deviation becomes smaller with increasing χ . With $\chi = 1600$ used for the main results shown in Fig. 3, the deviation remained within 0.8% of the initial value, at the largest time inspected in this work (t = 50). Next, we evaluate the correlation length $\xi(t) \equiv \{[1/\sigma^2 \Omega(t)] \sum_{\ell} \ell^2 C_2(\ell, t)\}^{1/2}$ with $\sigma^2 \approx 0.51$. The data exhibit the power law $\xi(t) \sim t^{2/3}$ [Fig. 6(b)] as expected from the KPZ scaling.

Using these results, we plot the rescaled two-point function $\tilde{C}_2(\ell, t) = [\xi(t)/\Omega(t)]C_2(\ell, t)$ in Fig. 6(c). The data are in good agreement with the Prähofer-Spohn exact solution $f_{\text{KPZ}}(u)$ with $u = \ell/\xi(t)$, confirming the earlier finding in Ref. [11] without adjustable parameters. However, to evaluate the parameter $\alpha_1(t)$ therefrom [Eq. (3) with $\alpha = \alpha_1(t)$], we must deal with the slight deviation from $f_{\text{KPZ}}(u)$ visible even for the largest χ we used. To this end, in the inset of Fig. 6(c), we plot $\tilde{\alpha}_1(\ell,t) \equiv \xi(t)^2 C_2(\ell,t) / \{2t^{2/3} f_{\text{KPZ}}[\ell/\xi(t)]\}.$ According to Eq. (3), this quantity is expected to take a constant value α in the long-time limit. Indeed, as t increases, $\tilde{\alpha}_1(\ell, t)$ develops a plateau. The deviation from the plateau gives a measure of the error in the estimate $\alpha_1(t)$. Specifically, we measure $\alpha_1^{\max}(t) \equiv \max_u \tilde{\alpha}_1(u, t)$ and $\alpha_1^{\min}(t) \equiv \min_{|u|<1} \tilde{\alpha}_1(u, t)$ [Fig. 6(d)] and use $\{ [\alpha_1^{\max}(t) +$ $\alpha_1^{\min}(t)]/2 \}$ as the estimate of $\alpha_1(t)$ and $\alpha_1^{\max}(t) - \alpha_1^{\min}(t)$ as its error bar. The results in Fig. 6(d) indeed show that $\alpha_1(t)$ tends to converge to a constant in the long-time and large- χ limit.

Next, we consider the quantities related to the magnetization transfer. First, its variance exhibits the KPZ scaling law, $Var[h(x, t)] \sim t^{2/3}$ [Fig. 6(e) inset]. From this, we evaluate $\alpha_2(t) \equiv \operatorname{Var}[h(0, t)]/(t^{2/3}\operatorname{Var}[BR])$ in Fig. 6(e), where the data converge to a constant value in the longtime limit. The KPZ scaling laws predict $\alpha_1(t) = \alpha_2(t) = a$ constant α [Eqs. (3) and (4)]. This is confirmed in Fig. 6(f), where the ratio $\alpha_2(t)/\alpha_1(t)$ approaches one in the longtime and large- χ limit. Finally, we plot the spatial and temporal correlation functions, $C_s(\ell, t)$ and $C_t(t_1, t_2)$, respectively, in the rescaled units in Figs. 6(g) and 6(h). They converge remarkably with increasing χ and show good agreement with the Airy₀ covariance $C_0(u)$ and the Ferrari-Spohn exact solution (6), respectively, without adjustable parameters. These results provide strong evidence that the KPZ scaling laws for the two-point quantities are valid in the quantum model too.