Hyperlinked references for the lecture in Les Houches School on KPZ "Introduction to the KPZ equation and its experimental aspects" (ver.2)

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